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Quantum corrections to the high-frequency thermal microfield in a dense plasma

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Abstract. The effects of the quantum diffraction corrections at high temperature ($k_BT \geqslant$ 1 Ryd) are quantitatively investigated for the high-frequency component of the thermal microfield in a dense plasma. We make use of the double expansion of the static pair correlation function with respect to the classical plasma parameter Λ and $\hbar\omega_p/k_BT$. Neutral-point and singly-charged-point cases are treated. Numerical data are plotted up to $v = 1.4$.

1. Introduction

For the purposes of Stark spectroscopic diagnostics in dense and hot inertial compressed plasmas, we considered at length (Held and Deutsch 1981, hereafter referred to as I, see also Deutsch and Gombert 1976, Gombert and Deutsch 1978, Gombert 1981) the classical dense plasma corrections to the thermal microfields arising from higher-order Λ^n terms in the static pair distribution function. Λ denotes the classical plasma parameter $e^2/k_BT\lambda_D$. The purpose of the present work is to put the emphasis on the specific high-temperature quantum effects arising from the uncertainty principle when $k_BT \geqslant 1$ Ryd, and the thermal wavelength gets larger than the Landau minimum impact parameter e^2/k_BT. A detailed analysis of the corresponding diffraction corrections has already been performed on the equilibrium pair distribution function $g_2(r)$, within the framework of the one-component-plasma (OCP) model. This accounts accurately for the thermal properties of the high-frequency component of the thermal microfield. It should be kept in mind that these corrections are surely completely negligible for the low-frequency component, driven by the ionic fluid, with a thermal wavelength much smaller than e^2/k_BT. The electron contribution to the ion screening is not expected to change significantly, as demonstrated below, when the above $\hbar$ corrections are considered.

2. Quantum one-component-plasma (OCP) model

2.1. General

We consider a spinless electron fluid in the presence of a neutralising and rigid background. The diffraction corrections may then be introduced in the simplest way

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(Deutsch *et al* 1981), through a temperature-dependent effective interaction

$$U_{ee}(r) = (e^2/r)(1 - e^{-cr}) \tag{1}$$

with $\lambda_{ee} = \hbar(\frac{1}{2}m_e k_B T)^{-1/2}$ and $c = \sqrt{2\pi}/\lambda_{ee}$. Equation (1) represents the so-called Kramers potential, which amounts to a high-temperature approximation ($\beta = (k_B T)^{-1}$)

$$\mathrm{Tr}\, e^{-\beta H} \sim e^{-\beta U_{ee}(r)} \qquad \beta \to 0 \tag{2}$$

of the two-body partition function by a Gibbs exponential, while retaining only the relative s states ($l = 0$) within an electron pair.

2.2. *Significant parameters*

The double expansion (Deutsch *et al* 1981) for the two-body static pair correlations with respect to Λ and $\hbar\omega_p/k_B T$, ω_p being the electron plasma frequency $(4\pi ne^2/m)^{1/2}$ remains meaningful provided that ($\lambda_D^2 = k_B T/4\pi ne^2$)

$$D \equiv 1 - 4/c^2\lambda_D^2 > 0 \qquad \Lambda \leqslant 1. \tag{3}$$

Introducing the well known Holtzmark distance (Griem 1974)

$$r_0(\mathrm{cm}) = \frac{0.62}{n^{1/3}(\mathrm{cm}^{-3})} \qquad \text{with} \quad \tfrac{4}{15}(2\pi)^{3/2} r_0^3 n = 1$$

the discrepancies with respect to the classical Λ-expansion of $g_2(r)$ are measured by

$$c\lambda_D = q/v^3 > 2 \tag{4}$$

where

$$v = \frac{r_0}{\lambda_D} = (2.9921\Lambda)^{1/3} = 8.98 \times 10^{-2}\left(\frac{n(\mathrm{cm}^{-3})}{T(\mathrm{K})}\right)^{1/2}$$

and

$$q = \tfrac{15}{2}\eta^{-1/2} = \frac{2980}{T_e^{1/2}(\mathrm{K})} \qquad \text{where} \quad \eta = 0.4\lambda_{ee}/\lambda_D.$$

As far as microfield computations are concerned, it has proved technically more adequate to replace the dimensionless quantum parameter $h\omega_p/k_B T$ by its equivalent $\eta = (\lambda_{ee} k_B T/e^2)^2$.

The classical limit ($\hbar = 0$) is recovered for $\lambda = \eta = 0$ with $c = q = \infty$ (Hooper 1966, 1968).

The (q, v) values considered in this work are displayed in table 1 (see also figure 1). They range as

$$0.04 \leqslant q \leqslant 8 \qquad 0.2 \leqslant v \leqslant 1.4 \tag{5}$$

so that $q > 2v^3$. This corresponds to $\Lambda \leqslant 1$ and a temperature domain

$$1.35 \times 10^5 < T \leqslant 5.97 \times 10^9\ \mathrm{K}$$

in between the hydrogen dissociation energy and the electron relativistic limit.

Here, we restrict to a hatched domain in figure 1 where the diffraction corrections dominate the symmetry ones associated with the Pauli principle. These latter are negligible provided

$$\eta \gg \chi = 0.2(\tfrac{4}{3}\pi n\lambda_{ee}^3)$$

Table 1. Quantum and v parameter values considered in this work.

$4/c^2\lambda_D^2$	q	v	$4/c^2\lambda_D^2$	q	v
4.00×10^{-6}	8.00	0.2	1.87×10^{-1}	8.00	1.2
2.56×10^{-4}	8.00	0.4		4.63	1.0
	1.00	0.2		2.37	0.8
2.92×10^{-3}	8.00	0.6		1.00	0.6
	2.37	0.4		2.96×10^{-1}	0.4
	2.96×10^{-1}	0.2		3.70×10^{-2}	0.2
1.64×10^{-2}	8.00	0.8	4.71×10^{-1}	8.00	1.4
	3.38	0.6		5.04	1.2
	1.00	0.4		2.92	1.0
	1.25×10^{-1}	0.2		1.49	0.8
6.25×10^{-2}	8.00	1.0		6.30×10^{-1}	0.6
	4.10	0.8		1.87×10^{-1}	0.4
	1.73	0.6		2.33×10^{-2}	0.2
	5.12×10^{-1}	0.4			
	6.40×10^{-2}	0.2			

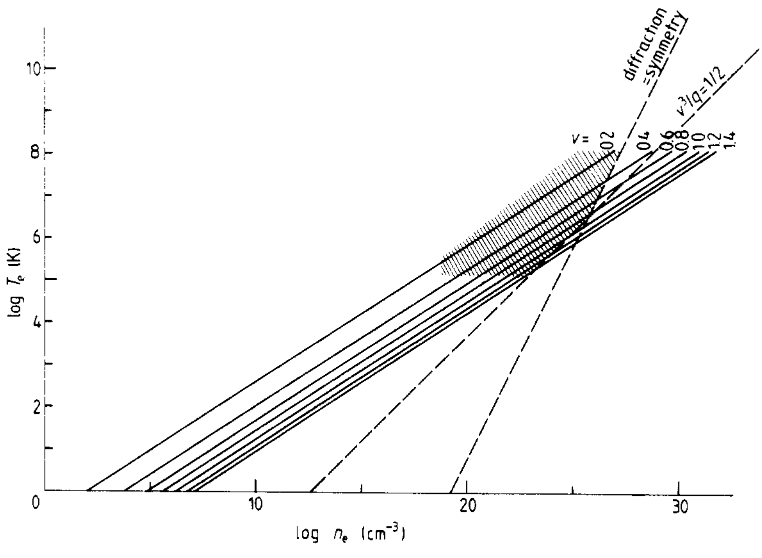

Figure 1. Log T–log n plot with the corresponding v values. The hatched domain is the area considered in this work.

i.e.

$$\log T(\mathrm{K}) \gg \log n(\mathrm{cm}^{-3}) - 20.$$

For instance, in the sun's interior one has

$$\Lambda = 0.059 \qquad v = 0.5 \qquad \eta = 0.24 \qquad \chi = 0.1$$

a situation which requires both η and χ corrections to the microfield distribution. This point will be taken up in a forthcoming article.

Pair distribution function

The microfield computation detailed is based on the $g_2(r)$ Λ expansion ($x = r/\lambda_D$) (Deutsch and Gombert 1976)

$$g_2(x) = \Lambda g_{2,\Lambda}(x) + \Lambda^2 g_{2,\Lambda}2(x) \tag{6}$$

explained through

$$g_{2,\Lambda}(x) = -D^{-1/2}\left(\frac{e^{-R_1x}}{x} - \frac{e^{-R_2x}}{x}\right) \tag{7}$$

and

$$g_{2,\Lambda^2}(x) = -\frac{1}{2D^2}\left\{\left([A_1(D^{1/2} + B - 1/4R_1^2) + C_1]\frac{e^{-R_1x}}{R_1x}\right.\right.$$
$$\left.\left. + (-A_2(D^{1/2} + B + 1/4R_2^2) + C_2)\frac{e^{-R_2x}}{R_2x}\right) - \left(\frac{A_1}{4R_1^2}e^{-R_1x} + \frac{A_2}{4R_2^2}e^{-R_2x}\right)\right\} \tag{8}$$

restricted to its $x \to \infty$ dominant contribution.

The corresponding parameters read

$$R_{\substack{1\\2}} = \frac{q}{v^3}\left\{\frac{1}{2}\left[1 \mp \left(1 - \frac{4v^6}{q^2}\right)^{1/2}\right]\right\}^{1/2}$$
$$A_{\substack{1\\2}} = \ln\left(\frac{(-)^{\substack{2\\1}}3(R_{\substack{1\\2}} + 2R_{\substack{2\\1}})R^2_{\substack{2\\1}}}{(2R_{\substack{2\\1}} - R_{\substack{1\\2}})(2R_{\substack{1\\2}} + R_{\substack{2\\1}})^2}\right) \qquad B = \frac{1}{R_2^2 - R_1^2} \tag{9}$$
$$C_{\substack{1\\2}} = \frac{(R_1 - R_2)^2}{2R^2_{\substack{1\\2}}}\left(\frac{1}{R_{\substack{2\\1}}(2R_{\substack{2\\1}} - R_{\substack{1\\2}})} + \frac{1}{3(2R_{\substack{1\\2}} + R_{\substack{2\\1}})(2R_{\substack{2\\1}} + R_{\substack{1\\2}})}\right)$$
$$D = 1 - 4v^6/q^2.$$

The classical $g_2(x)$ is recovered through

$$R_1 \sim 1, \qquad R_2 \gg 1, \qquad A_1 \sim \ln 3, \qquad A_2 \sim \ln 3/4R_2^2$$
$$B \ll 1, \qquad C_1 \sim \tfrac{1}{3}, \qquad C_2 \ll 1$$

with

$$-g_{2,\Lambda}(x) \sim e^{-x}/x$$

and

$$g_{2,\Lambda^2}(x) \sim -\tfrac{1}{8}[(\tfrac{4}{3} + 3\ln 3)\, e^{-x}/x - \ln 3\, e^{-x}].$$

3. Basic formalism

3.1. General

As in I, we analyse the Fourier transform of the high-frequency thermal microfield ($u = kE_0$, $E_0 = e/r_0^2$) under the form (Pfenning and Trefftz 1966)

$$F(u) = \exp\left(\sum_{p=1}^{\infty}\frac{n_e^p}{p!}h_p(u)\right)$$
$$\simeq \exp[n_e h_1(u) + \tfrac{1}{2}n_e^2 h_2(u)] \tag{10}$$

with (Held and Deutsch 1981)

$$n_e h_1(u) = -u^{3/2}\psi_1 \tag{11}$$

and

$$\psi_1 = \frac{15}{2\sqrt{2\pi}}\frac{1}{a^3}\int_0^\infty (1-j_0(z))g_1(x)x^2\,dx$$

where $z = a^2/x^2$ and $a^2 = ke/\lambda_D^2$. In the neutral case $g_1(x) = 1$ ($\psi_1 = 1$) while

$$g_1(x) = \exp\left(-\frac{\Lambda}{D^{1/2}}\frac{1}{x}(e^{-R_1x} - e^{-R_2x})\right)$$

at a singly-charged point.

The binary-correlated part reads ($\phi_i = \exp(i\boldsymbol{k}\cdot\boldsymbol{E}_i) - 1$)

$$\begin{aligned}\frac{n_e^2}{2}h_2(u) &= \iint \phi_1\phi_2 g_2(|\boldsymbol{r}_1 - \boldsymbol{r}_2|)\,d^3\boldsymbol{r}_1\,d^3\boldsymbol{r}_2\\ &= u^{3/2}[\psi_{2,\Lambda} + v^3\psi_{2,\Lambda^2}]\end{aligned} \tag{12}$$

with

$$\psi_{2,\Lambda} = -\frac{15}{2\sqrt{2\pi}}\frac{1}{a^3}\sum_l (-1)^l(2l+1)\chi_\Lambda^l \tag{13}$$

$$\psi_{2,\Lambda^2} = -\frac{1}{a^3}\sum_l (-)^l(2l+1)\chi_{\Lambda^2}^l \tag{14}$$

and

$$\chi_{\Lambda^i}^l = \int_0^\infty\int_0^{x_1} [j_l(z_1) - \delta_{l0}][j_l(z_2) - \delta_{l0}]\frac{f_{\Lambda^i}^l}{4\pi}x_1^2x_2^2\,dx_1\,dx_2 \tag{15}$$

where the $f_{\Lambda^i}^l$ will be determined below.

3.2. *First order*

The neutral point calculations are identical to those performed in the classical case (Held and Deutsch 1981), where

$$\psi_1 = 1 \qquad \text{and} \qquad n_e h_1(u) = -u^{3/2}.$$

At a singly charged point, one has to introduce

$$n_e h_1(u) = -u^{3/2}\psi_c(a)$$

$$\psi_c(a) = \frac{15}{2\sqrt{2\pi}}\frac{1}{a^3}\int_0^\infty [1 - j_0(z_1)]\exp\left(-\frac{\Lambda}{D^{1/2}}\frac{(e^{-R_1x} - e^{-R_2x})}{x}\right)x_1^2\,dx_1. \tag{16}$$

The corresponding f_Λ^i are

$$f_\Lambda^0 = \frac{4\pi}{D^{1/2}x_1x_2}\left(\frac{e^{-R_1x_1}}{R_1}\sinh(R_1x_2) - \frac{e^{-R_2x_1}}{R_2}\sinh(R_2x_2)\right) \tag{17}$$

$$f_\Lambda^1=\frac{4}{D^{1/2}x_1^2x_2^2}\left[\left(x_1+\frac{1}{R_1}\right)\frac{e^{-R_1x_1}}{R_1}\left(x_2\cosh(R_1x_2)-\frac{\sinh(R_1x_2)}{R_1}\right)\right.$$
$$\left.-\left(x_1+\frac{1}{R_2}\right)\frac{e^{-R_2x_1}}{R_2}\left(x_2\cosh(R_2x_2)-\frac{1}{R_2}\sinh(R_2x_2)\right)\right] \tag{18}$$

$$f_\Lambda^2=\frac{4\pi}{D^{1/2}x_1^3x_2^3}\left\{\left(x_1^2+\frac{3x_1}{R_1}+\frac{3}{R_1^2}\right)\frac{e^{-R_1x_2}}{R_1}\left[\left(x_2^2+\frac{3}{R_1^2}\right)\sinh(R_1x_2)-\frac{3x_2}{R_1}\cosh(R_1x_2)\right]\right.$$
$$\left.-\left(x_1^2+\frac{3x_1}{R_2}+\frac{3}{R_2^2}\right)\frac{e^{-R_2x_1}}{R_2}\left[\left(x_2^2+\frac{3}{R_2^2}\right)\sinh(R_2x_2)-\frac{3x_2}{R_2}\cosh(R_2x_2)\right]\right\}. \tag{19}$$

3.3. Second order

$h_2(u)$ is then obtained from

$$f_{\Lambda^2}^0=\frac{1}{2D^2}\frac{4\pi}{x_1x_2}\left[\!\!\left[\left(\left\{\left[A_1\left(D^{1/2}+B-\frac{1}{4R_1^2}\right)+C_1\right]\right.\right.\right.\right.$$
$$\left.\left.-\frac{A_1}{4R_1}\left(x_1+\frac{1}{R_1}\right)\right\}\sinh(R_1x_2)+\frac{A_1}{4R_1}x_2\cosh(R_1x_2)\right)\frac{e^{-R_1x_1}}{R_1^2}$$
$$+\left(\left\{\left[-A_2\left(D^{1/2}+B+\frac{1}{4R_2^2}\right)+C_2\right]-\frac{A_2}{4R_2}\left(x_1+\frac{1}{R_2}\right)\right\}\sinh(R_2x_2)\right.$$
$$\left.\left.+\frac{A_2x_2}{4R_2}\cosh(R_2x_2)\right)\frac{e^{-R_2x_1}}{R_2^2}\right]\!\!\right] \tag{20}$$

$$f_{\Lambda^2}^1=\frac{1}{2D^2}\frac{4\pi}{x_1^2x_2^2}\left[\!\!\left[\left(\left\{\left[A_1\left(D^{1/2}+B-\frac{1}{4R_1^2}\right)+C_1\right]\left(x_1+\frac{1}{R_1}\right)\right.\right.\right.\right.$$
$$\left.-\frac{A_1}{4R_1}\left(x_1^2+\frac{3}{R_1}x_1+\frac{3}{R_1^2}\right)\right\}x_2\cosh(R_1x_2)$$
$$-\frac{1}{R_1}\left\{\left[A_1\left(D^{1/2}+B-\frac{1}{4R_1^2}\right)+C_1\right]\right.$$
$$\left.\left.\times\left(x_1+\frac{1}{R_1}\right)-\frac{A_1}{4}\left(x_1x_2^2+\frac{1}{R_1}(x_1^2+x_2^2)+\frac{3}{R_1^2}x_1+\frac{3}{R_1^3}\right)\right\}\sinh(R_1x_2)\right)\frac{e^{-R_1x_1}}{R_1^2}$$
$$+\left(\left\{\left[-A_2\left(D^{1/2}+B+\frac{1}{4R_2^2}\right)+C_2\right]\left(x_1+\frac{1}{R_2}\right)\right.\right.$$
$$\left.-\frac{A_2}{4R_2}\left(x_1^2+\frac{3}{R_2}x_1+\frac{3}{R_2^2}\right)\right\}x_2$$
$$\times\cosh(R_2x_2)-\frac{1}{R_2}\left\{\left[-A_2\left(D^{1/2}+B+\frac{1}{4R_2^2}\right)+C_2\right]\left(x_1+\frac{1}{R_2}\right)\right.$$
$$\left.\left.\left.-\frac{A_2}{4}\left(x_1x_2^2+\frac{1}{R_2}(x_1^2+x_2^2)+\frac{3}{R_2^2}x_1+\frac{3}{R_2^3}\right)\right\}\sinh(R_2x_2)\right)\frac{e^{-R_2x_1}}{R_2^2}\right]\!\!\right] \tag{21}$$

and

$$f_{\Lambda^2}^2=\frac{1}{2D^2}\frac{4\pi}{x_1^3x_2^3}\left[\!\!\left[\left(\left\{\left[A_1\left(D^{1/2}+B-\frac{1}{4R_1^2}\right)+C_1\right]\left(x_1^2+\frac{3}{R_1}x_1+\frac{3}{R_1^2}\right)\left(x_2^2+\frac{3}{R_1^2}\right)\right.\right.\right.\right.$$

$$
-\frac{A_1}{4R_1}\left(x_1^3x_2^2+\frac{7}{R_1}x_1^2x_2^2+\frac{3}{R_1^2}x_1(x_1^2+6x_2^2)+\frac{18}{R_1^3}(x_1^2+x_2^2)+\frac{45}{R_1^4}x_1+\frac{45}{R_1^5}\right)\Bigg\}
$$

$$
\times\sinh(R_1x_2)-\frac{1}{R_1}\Bigg\{3\left[A_1\left(D^{1/2}+B-\frac{1}{4R_1^2}\right)+C_1\right]\left(x_1^2+\frac{3}{R_1}x_1+\frac{3}{R_1^2}\right)
$$

$$
-\frac{A_1}{4}\left(x_1^2x_2^2+\frac{3}{R_1}x_1(x_1^2+x_2^2)+\frac{3}{R_1^2}(6x_1^2+x_2^2)\right.
$$

$$
\left.+\frac{45}{R_1^3}x_1+\frac{45}{R_1^4}\right)\Bigg\}x_2\cosh(R_1x_2)\Bigg)\frac{e^{-R_1x_1}}{R_1^2}
$$

$$
+\Bigg(\Bigg\{\left[-A_2\left(D^{1/2}+B+\frac{1}{4R_2^2}\right)+C_2\right]\left(x_1^2+\frac{3}{R_2}x_1+\frac{3}{R_2^2}\right)\left(x_2^2+\frac{3}{R_2^2}\right)
$$

$$
-\frac{A_2}{4R_2}\left(x_1^3x_2^2+\frac{7}{R_2}x_1^2x_2^2+\frac{3}{R_2^2}x_1(x_1^2+6x_2^2)+\frac{18}{R_2^3}(x_1^2+x_2^2)+\frac{45}{R_2^4}x_1+\frac{45}{R_2^5}\right)\Bigg\}
$$

$$
\times\sinh(R_2x_2)-\frac{1}{R_2}\Bigg\{3\left[-A_2\left(D^{1/2}+B+\frac{1}{4R_2^2}\right)+C_2\right]\left(x_1^2+\frac{3}{R_2}x_1+\frac{3}{R_2^2}\right)
$$

$$
-\frac{A_2}{4}\left(x_1^2x_2^2+\frac{3x_1}{R_2}(x_1^2+x_2^2)+\frac{3}{R_2^2}(6x_1^2+x_2^2)\right.
$$

$$
\left.+\frac{45}{R_2^3}x_1+\frac{45}{R_2^4}\right)\Bigg\}x_2\cosh(R_2x_2)\Bigg)\frac{e^{-R_2x_1}}{R_2^2}\Bigg]. \tag{22}
$$

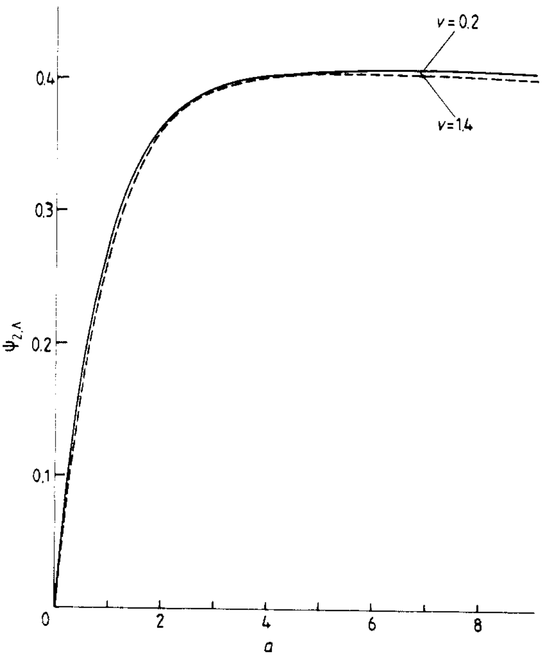

Figure 2. $\psi_{2,\Lambda}$ as a function of a.

4. Numerical results

$\psi_{2,\Lambda}$ shows up as a nearly v-independent function (figure 2) while ψ_{2,Λ^2} (figure 3) gets a minimum plateau at $v = 1.0$.

The high-frequency $H(\beta)$ are displayed in tables 2 and 3, for the neutral and the singly charged point respectively. The $\hbar$-corrected data (2nd row) are always located below their classical counterparts (1st row), with a maximum three per cent discrepancy.

The discrepancy increases with v. In the neutral case, it remains at 10^{-3} for $v \leq 0.6$, and both lines coalesce to the previous calculations which explains why we restrict table 2 to the range $0.8 \leq v \leq 1.4$.

In the singly charged case (table 3), the $\hbar$ corrections are relatively more significant, so we have to start the tabulation from $v = 0.2$.

The corresponding $H(\beta)$ profiles may be seen in figures 4 and 5. The diffraction corrected values appear mostly concentrated at the top of the $H(\beta)$ curves.

The $\hbar$-corrected data are always located below their classical counterparts with a maximum 3% discrepancy.

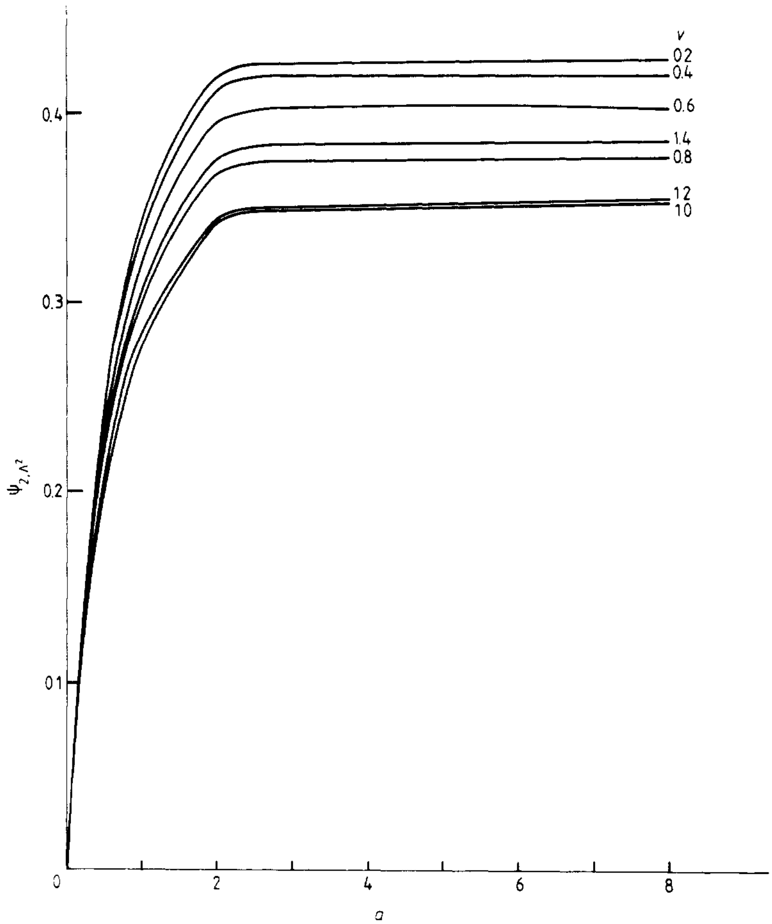

Figure 3. ψ_{2,Λ^2} as a function of a, for several v values.

Table 2. Probability distributions $H(\beta)$ of the high frequency at a neutral point for several values of v. The $\Lambda^2 - \hbar$ corrected diffraction results are the second entries, while the classical Λ^2 ones are given first.

β \ v	0.8	1.0	1.2	1.4
0.1	$0.914\,55 \times 10^{-2}$	$0.107\,72 \times 10^{-1}$	$0.129\,68 \times 10^{-1}$	$0.161\,49 \times 10^{-1}$
	$0.906\,56 \times 10^{-2}$	$0.105\,01 \times 10^{-1}$	$0.123\,65 \times 10^{-1}$	$0.154\,24 \times 10^{-1}$
0.2	$0.356\,51 \times 10^{-1}$	$0.418\,61 \times 10^{-1}$	$0.502\,31 \times 10^{-1}$	$0.622\,85 \times 10^{-1}$
	$0.353\,45 \times 10^{-1}$	$0.408\,32 \times 10^{-1}$	$0.479\,46 \times 10^{-1}$	$0.595\,45 \times 10^{-1}$
0.3	$0.768\,63 \times 10^{-1}$	$0.897\,87 \times 10^{-1}$	0.107 16	0.131 94
	$0.762\,20 \times 10^{-1}$	$0.876\,61 \times 10^{-1}$	0.102 47	0.126 35
0.4	0.128 80	0.149 39	0.176 96	0.215 78
	0.127 76	0.146 09	0.169 02	0.207 10
0.5	0.186 70	0.214 61	0.251 82	0.303 32
	0.185 26	0.210 12	0.242 10	0.291 96
0.6	0.245 66	0.279 37	0.324 08	0.384 77
	0.243 88	0.274 03	0.312 68	0.371 61
0.7	0.301 21	0.338 37	0.387 33	0.452 38
	0.299 18	0.332 58	0.375 22	0.438 59
0.8	0.349 74	0.387 59	0.473 03	0.501 25
	0.347 57	0.381 78	0.425 24	0.488 06
0.9	0.388 73	0.424 55	0.470 82	0.529 51
	0.386 55	0.419 14	0.460 28	0.517 97
1.0	0.416 86	0.448 36	0.488 35	0.537 92
	0.414 80	0.443 66	0.479 74	0.528 80
1.1	0.433 89	0.459 43	0.490 93	0.529 09
	0.432 04	0.455 64	0.484 63	0.522 82
1.2	0.440 44	0.459 15	0.480 92	0.506 75
	0.438 87	0.456 33	0.477 04	0.503 43
1.3	0.437 75	0.449 49	0.461 27	0.474 95
	0.436 51	0.447 61	0.459 63	0.474 39
1.4	0.427 39	0.432 65	0.434 98	0.437 48
	0.426 52	0.431 60	0.435 25	0.439 35
1.5	0.411 10	0.410 75	0.404 79	0.397 59
	0.410 60	0.410 39	0.406 52	0.401 41
1.6	0.390 54	0.385 71	0.372 97	0.357 78
	0.390 41	0.385 90	0.375 71	0.363 04
1.7	0.367 23	0.359 08	0.341 23	0.319 82
	0.367 44	0.359 68	0.344 56	0.326 02
1.8	0.342 45	0.332 05	0.310 79	0.284 80
	0.342 96	0.332 96	0.314 35	0.291 47
1.9	0.317 23	0.305 50	0.282 36	0.253 31
	0.318 01	0.306 65	0.285 90	0.260 03
2.0	0.292 37	0.280 01	0.256 34	0.225 51
	0.293 37	0.281 34	0.259 66	0.231 94
2.5	0.187 39	0.176 12	0.160 93	0.134 01
	0.188 76	0.177 97	0.163 08	0.140 66
3.0	0.120 44	0.110 83	0.104 95	$0.901\,59 \times 10^{-1}$
	0.121 18	0.112 55	0.107 52	$0.936\,59 \times 10^{-1}$
3.5	$0.804\,45 \times 10^{-1}$	$0.733\,29 \times 10^{-1}$	$0.693\,07 \times 10^{-1}$	$0.646\,52 \times 10^{-1}$
	$0.804\,32 \times 10^{-1}$	$0.746\,97 \times 10^{-1}$	$0.720\,38 \times 10^{-1}$	$0.652\,34 \times 10^{-1}$
4.0	$0.561\,49 \times 10^{-1}$	$0.517\,47 \times 10^{-1}$	$0.484\,37 \times 10^{-1}$	$0.460\,44 \times 10^{-1}$
	$0.559\,62 \times 10^{-1}$	$0.530\,02 \times 10^{-1}$	$0.507\,81 \times 10^{-1}$	$0.468\,75 \times 10^{-1}$
4.5	$0.408\,39 \times 10^{-1}$	$0.378\,58 \times 10^{-1}$	$0.353\,84 \times 10^{-1}$	$0.331\,37 \times 10^{-1}$
	$0.409\,08 \times 10^{-1}$	$0.387\,05 \times 10^{-1}$	$0.372\,47 \times 10^{-1}$	$0.345\,70 \times 10^{-1}$

Table 2. (continued)

β \ v	0.8	1.0	1.2	1.4
5.0	$0.307\,96\times10^{-1}$	$0.284\,41\times10^{-1}$	$0.266\,11\times10^{-1}$	$0.246\,86\times10^{-1}$
	$0.311\,47\times10^{-1}$	$0.293\,37\times10^{-1}$	$0.282\,64\times10^{-1}$	$0.262\,96\times10^{-1}$
6.0	$0.189\,93\times10^{-1}$	$0.176\,23\times10^{-1}$	$0.168\,25\times10^{-1}$	$0.153\,09\times10^{-1}$
	$0.194\,06\times10^{-1}$	$0.181\,44\times10^{-1}$	$0.174\,64\times10^{-1}$	$0.164\,43\times10^{-1}$
7.0	$0.125\,40\times10^{-1}$	$0.117\,57\times10^{-1}$	$0.110\,29\times10^{-1}$	$0.103\,45\times10^{-1}$
	$0.127\,49\times10^{-1}$	$0.122\,37\times10^{-1}$	$0.116\,47\times10^{-1}$	$0.110\,30\times10^{-1}$
8.0	$0.875\,90\times10^{-2}$	$0.835\,26\times10^{-2}$	$0.803\,13\times10^{-2}$	$0.734\,73\times10^{-2}$
	$0.892\,95\times10^{-2}$	$0.857\,77\times10^{-2}$	$0.823\,85\times10^{-2}$	$0.784\,45\times10^{-2}$
9.0	$0.658\,06\times10^{-2}$	$0.629\,92\times10^{-2}$	$0.579\,34\times10^{-2}$	$0.547\,33\times10^{-2}$
	$0.654\,99\times10^{-2}$	$0.630\,60\times10^{-2}$	$0.605\,36\times10^{-2}$	$0.578\,19\times10^{-2}$
10.0	$0.495\,53\times10^{-2}$	$0.474\,67\times10^{-2}$	$0.452\,64\times10^{-2}$	$0.432\,22\times10^{-2}$
	$0.497\,68\times10^{-2}$	$0.480\,38\times10^{-2}$	$0.461\,71\times10^{-2}$	$0.442\,36\times10^{-2}$
12.0	$0.309\,50\times10^{-2}$	$0.297\,89\times10^{-2}$	$0.284\,46\times10^{-2}$	$0.269\,72\times10^{-2}$
	$0.310\,74\times10^{-2}$	$0.301\,41\times10^{-2}$	$0.290\,83\times10^{-2}$	$0.279\,41\times10^{-2}$
14.0	$0.208\,58\times10^{-2}$	$0.210\,64\times10^{-2}$	$0.193\,21\times10^{-2}$	$0.183\,24\times10^{-2}$
	$0.209\,34\times10^{-2}$	$0.203\,87\times10^{-2}$	$0.197\,50\times10^{-2}$	$0.190\,48\times10^{-2}$
16.0	$0.148\,46\times10^{-2}$	$0.144\,05\times10^{-2}$	$0.138\,53\times10^{-2}$	$0.131\,72\times10^{-2}$
	$0.148\,95\times10^{-2}$	$0.145\,52\times10^{-2}$	$0.141\,46\times10^{-2}$	$0.136\,93\times10^{-2}$
18.0	$0.110\,12\times10^{-2}$	$0.107\,17\times10^{-2}$	$0.103\,40\times10^{-2}$	$0.986\,33\times10^{-3}$
	$0.110\,45\times10^{-2}$	$0.108\,18\times10^{-2}$	$0.105\,47\times10^{-2}$	$0.102\,41\times10^{-2}$
20.0	$0.843\,49\times10^{-3}$	$0.823\,01\times10^{-3}$	$0.796\,39\times10^{-3}$	$0.762\,06\times10^{-3}$
	$0.845\,81\times10^{-3}$	$0.830\,18\times10^{-3}$	$0.811\,32\times10^{-3}$	$0.789\,96\times10^{-3}$
22.0	$0.663\,09\times10^{-3}$	$0.648\,35\times10^{-3}$	$0.628\,98\times10^{-3}$	$0.603\,63\times10^{-3}$
	$0.664\,77\times10^{-3}$	$0.653\,60\times10^{-3}$	$0.640\,05\times10^{-3}$	$0.624\,63\times10^{-3}$
24.0	$0.532\,49\times10^{-3}$	$0.521\,59\times10^{-3}$	$0.507\,13\times10^{-3}$	$0.487\,99\times10^{-3}$
	$0.533\,75\times10^{-3}$	$0.525\,53\times10^{-3}$	$0.515\,51\times10^{-3}$	$0.504\,08\times10^{-3}$
26.0	$0.435\,31\times10^{-3}$	$0.427\,05\times10^{-3}$	$0.416\,01\times10^{-3}$	$0.401\,27\times10^{-3}$
	$0.436\,26\times10^{-3}$	$0.430\,07\times10^{-3}$	$0.422\,48\times10^{-3}$	$0.413\,81\times10^{-3}$
28.0	$0.361\,29\times10^{-3}$	$0.354\,90\times10^{-3}$	$0.346\,31\times10^{-3}$	$0.334\,75\times10^{-3}$
	$0.362\,03\times10^{-3}$	$0.357\,26\times10^{-3}$	$0.351\,40\times10^{-3}$	$0.344\,69\times10^{-3}$
30.0	$0.303\,78\times10^{-3}$	$0.298\,75\times10^{-3}$	$0.291\,95\times10^{-3}$	$0.282\,76\times10^{-3}$
	$0.304\,36\times10^{-3}$	$0.300\,62\times10^{-3}$	$0.296\,02\times10^{-3}$	$0.290\,73\times10^{-3}$
35.0	$0.206\,30\times10^{-3}$	$0.203\,37\times10^{-3}$	$0.199\,34\times10^{-3}$	$0.193\,84\times10^{-3}$
	$0.206\,65\times10^{-3}$	$0.204\,48\times10^{-3}$	$0.201\,79\times10^{-3}$	$0.198\,69\times10^{-3}$
40.0	$0.147\,61\times10^{-3}$	$0.145\,76\times10^{-3}$	$0.143\,21\times10^{-3}$	$0.139\,70\times10^{-3}$
	$0.147\,83\times10^{-3}$	$0.146\,47\times10^{-3}$	$0.144\,78\times10^{-3}$	$0.142\,84\times10^{-3}$
45.0	$0.109\,89\times10^{-3}$	$0.108\,67\times10^{-3}$	$0.106\,96\times10^{-3}$	$0.104\,60\times10^{-3}$
	$0.110\,04\times10^{-3}$	$0.109\,14\times10^{-3}$	$0.108\,02\times10^{-3}$	$0.106\,73\times10^{-3}$
50.0	$0.844\,12\times10^{-4}$	$0.835\,61\times10^{-4}$	$0.823\,75\times10^{-4}$	$0.807\,20\times10^{-4}$
	$0.845\,14\times10^{-4}$	$0.838\,93\times10^{-4}$	$0.831\,20\times10^{-4}$	$0.822\,23\times10^{-4}$
60.0	$0.534\,90\times10^{-4}$	$0.530\,39\times10^{-4}$	$0.524\,04\times10^{-4}$	$0.515\,13\times10^{-4}$
	$0.535\,45\times10^{-4}$	$0.523\,17\times10^{-4}$	$0.528\,08\times10^{-4}$	$0.523\,32\times10^{-4}$
70.0	$0.363\,78\times10^{-4}$	$0.361\,14\times10^{-4}$	$0.357\,41\times10^{-4}$	$0.352\,14\times10^{-4}$
	$0.364\,10\times10^{-4}$	$0.362\,19\times10^{-4}$	$0.359\,80\times10^{-4}$	$0.357\,02\times10^{-4}$
80.0	$0.260\,53\times10^{-4}$	$0.258\,87\times10^{-4}$	$0.256\,51\times10^{-4}$	$0.253\,17\times10^{-4}$
	$0.260\,73\times10^{-4}$	$0.259\,53\times10^{-4}$	$0.258\,03\times10^{-4}$	$0.256\,28\times10^{-4}$
90.0	$0.194\,08\times10^{-4}$	$0.192\,98\times10^{-4}$	$0.191\,41\times10^{-4}$	$0.189\,18\times10^{-4}$
	$0.194\,22\times10^{-4}$	$0.193\,43\times10^{-4}$	$0.192\,43\times10^{-4}$	$0.191\,27\times10^{-4}$
100.0	$0.149\,15\times10^{-4}$	$0.148\,39\times10^{-4}$	$0.147\,29\times10^{-4}$	$0.145\,74\times10^{-4}$
	$0.149\,24\times10^{-4}$	$0.148\,70\times10^{-4}$	$0.148\,01\times10^{-4}$	$0.147\,20\times10^{-4}$

Table 3. Probability distribution $H(\beta)$ of the high frequency at a charged point for several values of v. Same presentation of the high frequency as in table 1.

β \ v	0.2	0.4	0.6	0.8	1.0	1.2	1.4
0.1	$0.555\,05\times10^{-2}$	$0.717\,87\times10^{-2}$	$0.905\,28\times10^{-2}$	$0.112\,83\times10^{-1}$	$0.142\,10\times10^{-1}$	$0.181\,56\times10^{-1}$	$0.238\,13\times10^{-1}$
	$0.555\,07\times10^{-2}$	$0.717\,82\times10^{-2}$	$0.904\,17\times10^{-2}$	$0.111\,81\times10^{-1}$	$0.138\,38\times10^{-1}$	$0.172\,87\times10^{-1}$	$0.227\,91\times10^{-1}$
0.2	$0.218\,17\times10^{-1}$	$0.281\,17\times10^{-1}$	$0.353\,44\times10^{-1}$	$0.439\,01\times10^{-1}$	$0.550\,83\times10^{-1}$	$0.701\,15\times10^{-1}$	$0.915\,12\times10^{-1}$
	$0.218\,18\times10^{-1}$	$0.281\,15\times10^{-1}$	$0.353\,02\times10^{-1}$	$0.435\,10\times10^{-1}$	$0.536\,71\times10^{-1}$	$0.668\,28\times10^{-1}$	$0.876\,46\times10^{-1}$
0.3	$0.476\,86\times10^{-1}$	$0.610\,94\times10^{-1}$	0.76393×10^{-1}	$0.943\,63\times10^{-1}$	0.117 66	0.148 82	0.192 68
	$0.476\,88\times10^{-1}$	$0.610\,89\times10^{-1}$	$0.763\,04\times10^{-1}$	$0.935\,42\times10^{-1}$	0.114 75	0.142 09	0.184 76
0.4	$0.814\,21\times10^{-1}$	0.103 47	0.128 44	0.157 45	0.194 62	0.244 01	0.312 44
	$0.814\,25\times10^{-1}$	0.103 46	0.128 30	0.156 13	0.190 06	0.233 52	0.300 08
0.5	0.120 84	0.152 00	0.186 96	0.226 99	0.277 50	0.344 03	0.434 36
	0.120 84	0.151 99	0.186 76	0.225 17	0.271 43	0.330 19	0.418 01
0.6	0.163 50	0.203 17	0.247 18	0.296 72	0.357 96	0.437 72	0.543 47
	0.163 51	0.203 16	0.246 94	0.294 47	0.350 78	0.421 57	0.524 24
0.7	0.206 94	0.253 58	0.304 66	0.361 01	0.428 89	0.516 07	0.628 48
	0.206 95	0.253 56	0.304 39	0.358 45	0.421 16	0.498 98	0.607 85
0.8	0.248 83	0.300 21	0.355 68	0.415 48	0.485 19	0.573 12	0.682 97
	0.248 84	0.300 19	0.355 39	0.412 76	0.477 51	0.556 52	0.662 48
0.9	0.287 19	0.340 68	0.397 52	0.457 27	0.524 05	0.606 31	0.705 46
	0.287 20	0.340 66	0.397 24	0.454 24	0.516 95	0.591 14	0.686 44
1.0	0.320 43	0.373 33	0.428 61	0.485 08	0.544 89	0.616 14	0.698 52
	0.320 44	0.373 32	0.428 35	0.428 48	0.538 76	0.603 86	0.681 95
1.1	0.347 47	0.397 29	0.448 40	0.499 00	0.548 95	0.605 44	0.667 49
	0.347 48	0.397 28	0.448 17	0.496 66	0.543 98	0.596 19	0.653 95
1.2	0.367 73	0.412 37	0.457 28	0.500 20	0.538 68	0.578 46	0.618 99
	0.367 74	0.412 36	0.457 10	0.498 19	0.534 93	0.572 26	0.608 68
1.3	0.381 06	0.418 97	0.456 32	0.490 54	0.517 23	0.540 01	0.559 77
	0.381 07	0.418 97	0.456 18	0.488 94	0.514 58	0.536 53	0.552 58
1.4	0.387 71	0.417 94	0.447 01	0.472 28	0.487 83	0.494 71	0.495 81
	0.387 71	0.417 94	0.446 92	0.471 11	0.486 13	0.493 45	0.491 41
1.5	0.388 23	0.410 40	0.431 08	0.447 70	0.453 51	0.446 56	0.431 86
	0.388 23	0.410 40	0.431 03	0.446 97	0.452 54	0.446 88	0.429 82
1.6	0.383 36	0.397 58	0.410 25	0.418 96	0.416 78	0.398 64	0.371 32
	0.383 35	0.397 58	0.410 24	0.418 64	0.416 35	0.399 96	0.371 14

Table 3. (continued)

β \ v	0.2	0.4	0.6	0.8	1.0	1.2	1.4
1.7	0.373 98	0.380 76	0.386 13	0.387 91	0.379 61	0.353 13	0.316 32
	0.373 97	0.380 76	0.386 15	0.387 99	0.379 56	0.354 92	0.317 49
1.8	0.360 99	0.361 13	0.360 14	0.356 09	0.343 41	0.311 37	0.267 92
	0.360 98	0.361 13	0.360 17	0.356 52	0.343 63	0.313 22	0.269 97
1.9	0.345 29	0.339 73	0.333 42	0.324 66	0.309 08	0.273 99	0.226 39
	0.345 28	0.339 74	0.333 46	0.325 40	0.309 50	0.275 62	0.228 89
2.0	0.327 69	0.317 48	0.306 88	0.294 47	0.277 14	0.241 12	0.191 47
	0.327 68	0.317 48	0.306 93	0.295 46	0.277 74	0.242 38	0.194 03
2.5	0.232 89	0.212 82	0.193 50	0.174 64	0.156 29	0.132 48	$0.971\,45\times10^{-1}$
	0.232 87	0.212 82	0.193 51	0.176 04	0.157 61	0.132 75	0.105 24
3.0	0.155 70	0.138 09	0.121 08	0.104 86	$0.882\,17\times10^{-1}$	$0.773\,00\times10^{-1}$	$0.551\,35\times10^{-1}$
	0.155 69	0.138 09	0.121 01	0.105 51	$0.895\,50\times10^{-1}$	$0.776\,22\times10^{-1}$	$0.596\,60\times10^{-1}$
3.5	0.104 05	$0.911\,52\times10^{-1}$	$0.785\,08\times10^{-1}$	$0.661\,62\times10^{-1}$	$0.532\,80\times10^{-1}$	$0.446\,92\times10^{-1}$	$0.320\,71\times10^{-1}$
	0.104 04	$0.911\,35\times10^{-1}$	$0.783\,50\times10^{-1}$	$0.659\,09\times10^{-1}$	$0.543\,95\times10^{-1}$	$0.477\,50\times10^{-1}$	$0.362\,17\times10^{-1}$
4.0	$0.713\,90\times10^{-1}$	$0.623\,19\times10^{-1}$	$0.531\,76\times10^{-1}$	$0.440\,28\times10^{-1}$	$0.353\,13\times10^{-1}$	$0.271\,87\times10^{-1}$	$0.200\,00\times10^{-1}$
	$0.713\,89\times10^{-1}$	$0.622\,85\times10^{-1}$	$0.528\,82\times10^{-1}$	$0.436\,17\times10^{-1}$	$0.356\,25\times10^{-1}$	$0.296\,87\times10^{-1}$	$0.231\,25\times10^{-1}$
4.5	$0.507\,00\times10^{-1}$	$0.443\,05\times10^{-1}$	$0.376\,15\times10^{-1}$	$0.307\,58\times10^{-1}$	$0.243\,06\times10^{-1}$	$0.176\,92\times10^{-1}$	$0.130\,36\times10^{-1}$
	$0.507\,04\times10^{-1}$	$0.442\,87\times10^{-1}$	$0.371\,30\times10^{-1}$	$0.307\,31\times10^{-1}$	$0.244\,43\times10^{-1}$	$0.200\,20\times10^{-1}$	$0.158\,30\times10^{-1}$
5.0	$0.372\,46\times10^{-1}$	$0.326\,80\times10^{-1}$	$0.277\,91\times10^{-1}$	$0.224\,23\times10^{-1}$	$0.171\,34\times10^{-1}$	$0.119\,85\times10^{-1}$	$0.858\,65\times10^{-2}$
	$0.372\,52\times10^{-1}$	$0.327\,01\times10^{-1}$	$0.270\,69\times10^{-1}$	$0.227\,87\times10^{-1}$	$0.173\,22\times10^{-1}$	$0.141\,32\times10^{-1}$	$0.110\,91\times10^{-1}$
6.0	$0.219\,17\times10^{-1}$	$0.194\,36\times10^{-1}$	$0.165\,96\times10^{-1}$	$0.130\,85\times10^{-1}$	$0.970\,82\times10^{-2}$	$0.612\,37\times10^{-2}$	$0.387\,88\times10^{-2}$
	$0.219\,25\times10^{-1}$	$0.194\,66\times10^{-1}$	$0.165\,74\times10^{-1}$	$0.129\,18\times10^{-1}$	$0.985\,47\times10^{-2}$	$0.782\,48\times10^{-2}$	$0.612\,37\times10^{-2}$
7.0	$0.141\,02\times10^{-1}$	$0.125\,27\times10^{-1}$	$0.103\,20\times10^{-1}$	$0.818\,01\times10^{-2}$	$0.596\,10\times10^{-2}$	$0.354\,82\times10^{-2}$	$0.206\,75\times10^{-2}$
	$0.141\,10\times10^{-1}$	$0.124\,84\times10^{-1}$	$0.106\,76\times10^{-1}$	$0.833\,43\times10^{-2}$	$0.623\,00\times10^{-2}$	$0.489\,96\times10^{-2}$	$0.377\,96\times10^{-2}$
8.0	$0.968\,76\times10^{-2}$	$0.850\,53\times10^{-2}$	$0.704\,46\times10^{-2}$	$0.539\,52\times10^{-2}$	$0.395\,43\times10^{-2}$	$0.228\,20\times10^{-2}$	$0.115\,26\times10^{-2}$
	$0.969\,45\times10^{-2}$	$0.847\,53\times10^{-2}$	$0.734\,14\times10^{-2}$	$0.575\,49\times10^{-2}$	$0.426\,52\times10^{-2}$	$0.323\,89\times10^{-2}$	$0.254\,12\times10^{-2}$
9.0	$0.698\,94\times10^{-2}$	$0.608\,09\times10^{-2}$	$0.496\,02\times10^{-2}$	$0.395\,33\times10^{-2}$	$0.290\,81\times10^{-2}$	$0.157\,50\times10^{-2}$	$0.653\,66\times10^{-2}$
	$0.699\,53\times10^{-2}$	$0.611\,35\times10^{-2}$	$0.529\,52\times10^{-2}$	$0.417\,11\times10^{-2}$	$0.308\,40\times10^{-2}$	$0.229\,90\times10^{-2}$	$0.180\,04\times10^{-2}$
10.0	$0.524\,34\times10^{-2}$	$0.465\,81\times10^{-2}$	$0.392\,81\times10^{-2}$	$0.303\,51\times10^{-2}$	$0.207\,58\times10^{-2}$	$0.115\,20\times10^{-2}$	$0.401\,63\times10^{-3}$
	$0.524\,87\times10^{-2}$	$0.465\,04\times10^{-2}$	$0.396\,17\times10^{-2}$	$0.313\,58\times10^{-2}$	$0.232\,04\times10^{-2}$	$0.171\,46\times10^{-2}$	$0.131\,75\times10^{-2}$
12.0	$0.320\,13\times10^{-2}$	$0.282\,81\times10^{-2}$	$0.238\,05\times10^{-2}$	$0.184\,95\times10^{-2}$	$0.127\,07\times10^{-2}$	$0.691\,95\times10^{-3}$	$0.206\,69\times10^{-3}$
	$0.320\,51\times10^{-2}$	$0.282\,54\times10^{-2}$	$0.240\,60\times10^{-2}$	$0.192\,10\times10^{-2}$	$0.143\,01\times10^{-2}$	$0.105\,35\times10^{-2}$	$0.800\,73\times10^{-3}$

14.0	0.212 14 × 10^{-2}	0.186 15 × 10^{-2}	0.156 16 × 10^{-2}	0.121 83 × 10^{-2}	0.843 11 × 10^{-3}	0.459 15 × 10^{-3}	0.129 84 × 10^{-3}
	0.212 43 × 10^{-2}	0.186 10 × 10^{-2}	0.158 15 × 10^{-2}	0.127 18 × 10^{-2}	0.954 32 × 10^{-3}	0.706 70 × 10^{-3}	0.539 73 × 10^{-3}
16.0	0.149 03 × 10^{-2}	0.129 83 × 10^{-2}	0.108 44 × 10^{-2}	0.848 59 × 10^{-3}	0.591 70 × 10^{-3}	0.324 71 × 10^{-3}	0.925 85 × 10^{-4}
	0.149 26 × 10^{-2}	0.129 88 × 10^{-2}	0.110 03 × 10^{-2}	0.890 14 × 10^{-3}	0.673 36 × 10^{-3}	0.502 60 × 10^{-3}	0.387 66 × 10^{-3}
18.0	0.109 38 × 10^{-2}	0.945 77 × 10^{-3}	0.786 00 × 10^{-3}	0.616 38 × 10^{-3}	0.432 93 × 10^{-3}	0.240 07 × 10^{-3}	0.708 51 × 10^{-4}
	0.109 56 × 10^{-2}	0.946 66 × 10^{-3}	0.798 96 × 10^{-3}	0.649 65 × 10^{-3}	0.495 26 × 10^{-3}	0.372 84 × 10^{-3}	0.290 75 × 10^{-3}
20.0	0.830 47 × 10^{-3}	0.712 72 × 10^{-3}	0.589 12 × 10^{-3}	0.462 64 × 10^{-3}	0.327 15 × 10^{-3}	0.183 45 × 10^{-3}	0.565 05 × 10^{-4}
	0.831 98 × 10^{-3}	0.713 80 × 10^{-3}	0.599 92 × 10^{-3}	0.489 91 × 10^{-3}	0.376 22 × 10^{-3}	0.285 61 × 10^{-3}	0.225 13 × 10^{-3}
22.0	0.647 93 × 10^{-3}	0.551 91 × 10^{-3}	0.453 60 × 10^{-3}	0.356 53 × 10^{-3}	0.253 69 × 10^{-3}	0.143 84 × 10^{-3}	0.462 59 × 10^{-4}
	0.649 20 × 10^{-3}	0.553 05 × 10^{-3}	0.462 74 × 10^{-3}	0.379 31 × 10^{-3}	0.293 27 × 10^{-3}	0.224 43 × 10^{-3}	0.178 68 × 10^{-3}
24.0	0.516 89 × 10^{-3}	0.437 02 × 10^{-3}	0.357 06 × 10^{-3}	0.280 77 × 10^{-3}	0.200 93 × 10^{-3}	0.115 14 × 10^{-3}	0.385 65 × 10^{-4}
	0.517 97 × 10^{-3}	0.438 15 × 10^{-3}	0.364 89 × 10^{-3}	0.300 10 × 10^{-3}	0.233 52 × 10^{-3}	0.180 05 × 10^{-3}	0.144 67 × 10^{-3}
26.0	0.420 07 × 10^{-3}	0.352 55 × 10^{-3}	0.286 31 × 10^{-3}	0.225 15 × 10^{-3}	0.161 99 × 10^{-3}	0.937 52 × 10^{-4}	0.325 90 × 10^{-4}
	0.421 00 × 10^{-3}	0.353 65 × 10^{-3}	0.293 10 × 10^{-3}	0.241 77 × 10^{-3}	0.189 27 × 10^{-3}	0.146 96 × 10^{-3}	0.119 08 × 10^{-3}
28.0	0.346 80 × 10^{-3}	0.288 94 × 10^{-3}	0.233 20 × 10^{-3}	0.183 35 × 10^{-3}	0.132 57 × 10^{-3}	0.774 48 × 10^{-4}	0.278 40 × 10^{-4}
	0.347 61 × 10^{-3}	0.289 98 × 10^{-3}	0.239 15 × 10^{-3}	0.197 80 × 10^{-3}	0.155 73 × 10^{-3}	0.121 72 × 10^{-3}	0.993 94 × 10^{-4}
30.0	0.290 18 × 10^{-3}	0.240 03 × 10^{-3}	0.192 52 × 10^{-3}	0.151 30 × 10^{-3}	0.109 90 × 10^{-3}	0.647 73 × 10^{-4}	0.239 96 × 10^{-4}
	0.290 89 × 10^{-3}	0.241 92 × 10^{-3}	0.197 77 × 10^{-3}	0.163 98 × 10^{-3}	0.129 81 × 10^{-3}	0.102 09 × 10^{-3}	0.839 58 × 10^{-4}
35.0	0.194 99 × 10^{-3}	0.158 47 × 10^{-3}	0.125 08 × 10^{-3}	0.981 18 × 10^{-4}	0.720 25 × 10^{-4}	0.432 96 × 10^{-4}	0.170 83 × 10^{-4}
	0.195 53 × 10^{-3}	0.159 32 × 10^{-3}	0.129 06 × 10^{-3}	0.107 59 × 10^{-3}	0.862 50 × 10^{-4}	0.688 01 × 10^{-4}	0.574 62 × 10^{-4}
40.0	0.138 26 × 10^{-3}	0.110 45 × 10^{-3}	0.857 61 × 10^{-4}	0.670 88 × 10^{-4}	0.497 13 × 10^{-4}	0.304 03 × 10^{-4}	0.126 05 × 10^{-4}
	0.138 67 × 10^{-3}	0.111 18 × 10^{-3}	0.888 77 × 10^{-4}	0.744 45 × 10^{-4}	0.603 76 × 10^{-4}	0.487 72 × 10^{-4}	0.412 68 × 10^{-4}
45.0	0.102 11 × 10^{-3}	0.802 21 × 10^{-4}	0.612 55 × 10^{-4}	0.477 55 × 10^{-4}	0.356 94 × 10^{-4}	0.221 68 × 10^{-4}	0.956 54 × 10^{-5}
	0.102 44 × 10^{-3}	0.808 42 × 10^{-4}	0.637 65 × 10^{-4}	0.536 38 × 10^{-4}	0.439 80 × 10^{-4}	0.359 57 × 10^{-4}	0.307 54 × 10^{-4}
50.0	0.778 73 × 10^{-4}	0.601 85 × 10^{-4}	0.451 78 × 10^{-4}	0.350 82 × 10^{-4}	0.264 37 × 10^{-4}	0.166 51 × 10^{-4}	0.742 69 × 10^{-5}
	0.781 44 × 10^{-4}	0.607 21 × 10^{-4}	0.472 44 × 10^{-4}	0.398 97 × 10^{-4}	0.330 59 × 10^{-4}	0.273 01 × 10^{-4}	0.236 00 × 10^{-4}
60.0	0.487 22 × 10^{-4}	0.364 74 × 10^{-4}	0.264 31 × 10^{-4}	0.203 34 × 10^{-4}	0.155 65 × 10^{-4}	0.100 56 × 10^{-4}	0.472 69 × 10^{-5}
	0.489 13 × 10^{-4}	0.368 82 × 10^{-4}	0.279 03 × 10^{-4}	0.237 36 × 10^{-4}	0.200 69 × 10^{-4}	0.169 01 × 10^{-4}	0.148 68 × 10^{-4}
70.0	0.327 66 × 10^{-4}	0.237 80 × 10^{-4}	0.166 07 × 10^{-4}	0.126 36 × 10^{-4}	0.982 31 × 10^{-5}	0.649 70 × 10^{-5}	0.318 11 × 10^{-5}
	0.329 09 × 10^{-4}	0.241 01 × 10^{-4}	0.177 11 × 10^{-4}	0.151 71 × 10^{-4}	0.130 83 × 10^{-4}	0.112 19 × 10^{-4}	0.100 21 × 10^{-4}
80.0	0.232 29 × 10^{-4}	0.163 54 × 10^{-4}	0.109 84 × 10^{-4}	0.825 19 × 10^{-5}	0.651 77 × 10^{-5}	0.440 97 × 10^{-5}	0.223 31 × 10^{-5}
	0.233 39 × 10^{-4}	0.166 12 × 10^{-4}	0.118 43 × 10^{-4}	0.102 14 × 10^{-4}	0.898 54 × 10^{-5}	0.783 93 × 10^{-5}	0.709 87 × 10^{-5}
90.0	0.171 44 × 10^{-4}	0.117 13 × 10^{-4}	0.754 75 × 10^{-5}	0.558 76 × 10^{-5}	0.448 89 × 10^{-5}	0.310 69 × 10^{-5}	0.162 00 × 10^{-5}
	0.172 32 × 10^{-4}	0.119 26 × 10^{-4}	0.823 56 × 10^{-5}	0.715 27 × 10^{-5}	0.642 11 × 10^{-5}	0.569 71 × 10^{-5}	0.522 50 × 10^{-5}
100.0	0.130 60 × 10^{-4}	0.866 12 × 10^{-5}	0.533 92 × 10^{-5}	0.388 62 × 10^{-5}	0.318 06 × 10^{-5}	0.225 37 × 10^{-5}	0.120 66 × 10^{-5}
	0.131 32 × 10^{-4}	0.883 94 × 10^{-5}	0.590 30 × 10^{-5}	0.516 43 × 10^{-5}	0.473 43 × 10^{-5}	0.427 09 × 10^{-5}	0.396 44 × 10^{-5}

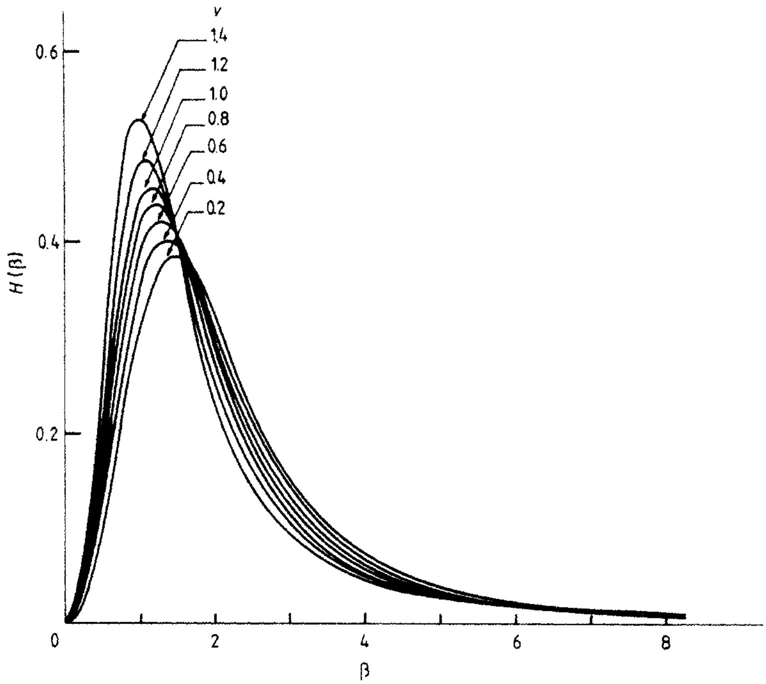

Figure 4. High-frequency $\hbar$-corrected $H(\beta)$ for $q=8$ at a neutral point.

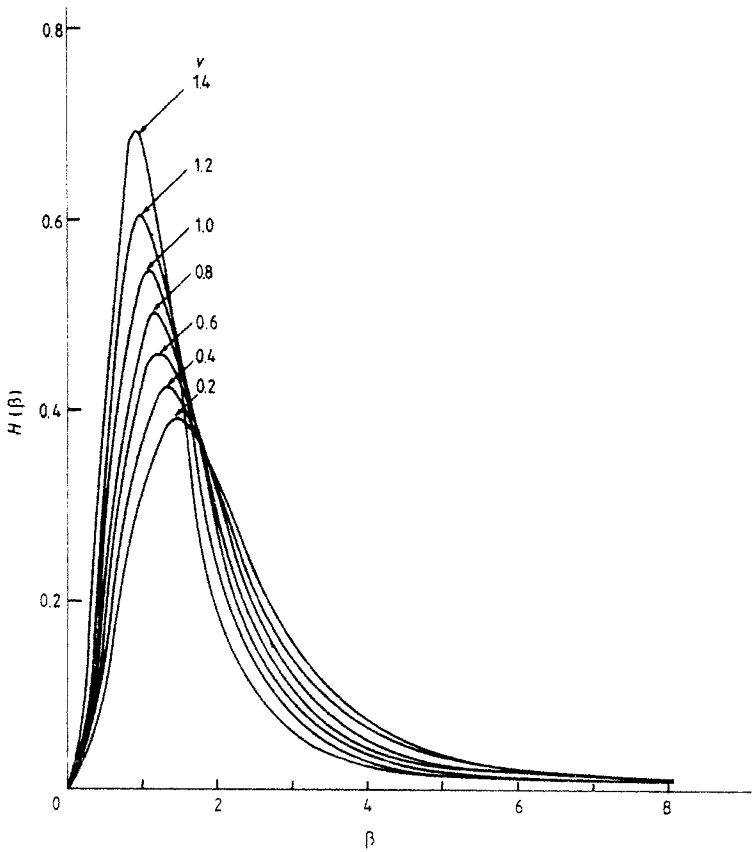

Figure 5. Same as in figure 4 for a singly charged point.

This discrepancy increases with v. In the neutral case, it remains at 10^{-3} for $v \leqslant 0.6$, and both lines coalesce to the previous calculations.

In the singly charged case, the h corrections are relatively more significant.

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